Drag Coefficient of a Spherical Particle Attached on the Flat Surface

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ABSTRACT

The previous analytical solution for the drag coefficient \( C_d \) for a spherical particle attached on the flat surface, which was derived by O’Neill (1968), is only valid in the creeping flow conditions. It is important to extend O’Neill’s formula to cover a wide range of particle Reynolds number \( Re_p \). In this study, the drag coefficient was calculated numerically to cover \( Re_p \) from 0.1 to 250. For a particle suspended in the air, an empirical drag coefficient exists, which is defined as \( C_d = f \times 24/Re_p \), where \( f \) is a correction factor depending on \( Re_p \). The applicability of the correction factor \( f \) for O’Neill’s analytical equation for the spherical particle attached on the flat surface for \( Re_p = 0.1 \) to 250 was examined in this study.

Keywords: Drag coefficient; Boundary layer; Numerical simulation; Reynolds number.

INTRODUCTION

Particle re-entrainment or detachment from solid surfaces is important in various engineering and environmental problems, such as surface cleaning, fugitive dust generation from unpaved surfaces and powder dispersion. Extensive literature has been published related to particle detachment from surfaces (Sehmel, 1980; Nicholson, 1988; Tsai et al., 1991a, b; Ziskind et al. 1995; Chiou and Tsai, 2001; Tsai and Chang, 2002; Tsai et al., 2003; Ziskind, 2006; Ibrahim et al., 2008; Gradon, 2009). The formation of a visible dust devil vortex depends on the presence of dust particles and the surface friction (Gu et al., 2010). There are three re-entrainment modes, namely direct lift-off, sliding and rolling. Among them rolling provides the least resistance for incipient detachment as compared to direct lift-off and rolling (Ibrahim et al., 2008). To determine if the particle detachment from the surfaces occurs, it is essential to calculate particle drag force accurately.

The previous analytical solution for the drag coefficient \( C_d \) for a spherical particle attached on the flat surface, which is only valid in the creeping flow conditions, was derived by O’Neill (1968) as

\[
C_d = 1.7009 \frac{24}{Re_p} \quad (1)
\]

where \( Re_p \) is the particle Reynolds number defined as

\[
Re_p = \frac{\rho U_p D_p}{\mu} \quad (2)
\]

where \( \rho \), \( U_p \), \( D_p \), and \( \mu \) are the density of air (kg/m\(^3\)), the velocity at the center of the particle (m/s), the particle diameter (m) and the air viscosity (kg/m-s), respectively. For \( 0.1 \leq Re_p \leq 250 \), Sweeney and Finlay (2007) developed an empirical drag coefficient at the plate Reynolds number \( Re_x \) of 32,400 based on the numerical simulation as

\[
C_d = \frac{40.812}{Re_p} \left[ 1 + \frac{0.2817}{Re_p^{0.57}} \text{arcsinh}(0.238Re_p) \right] \quad (3)
\]

The plate Reynolds number is defined as

\[
Re_x = \frac{\rho U_x x}{\mu} \quad (4)
\]

where \( U_x \) is the free-stream velocity (m/s), \( x \) is the distance from the leading edge (m). Note that the inverse hyperbolic sine term in Eq. (3) was mistaken for inverse sine in Sweeney and Finlay (2007) (Martinez et al. 2009). Ibrahim et al. (2008) used the following formula to calculate the drag force for the particle attached on the surface

\[
C_d = 1.7009 \frac{24}{Re_p} \left[ 1 + 3Re_p / 16 + 0.0079Re_p^2 + 9Re_p^2 \ln(Re_p) / 160 \right] \quad \text{for } Re_p < 8 \quad (5)
\]

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where the third term on the right hand side is the correction factor obtained from Ockendon and Evans (1972).

Up to now, very few studies on the drag coefficient of a spherical particle attached on the flat surface in a wide range of particle Reynolds number are available. It is desirable to have an accurate formula to predict the drag coefficient in non-creeping flow conditions. The objective of this study is to determine such drag coefficient numerically for $0.1 \leq \text{Re}_p \leq 250$.

For a particle suspended in the air, an empirical drag coefficient exists, which is defined as (Willeke and Baron, 1993)

$$C_d = f \frac{24}{\text{Re}_p}$$

(6)

where $f$ is the correction factor expressed as

$$f = \begin{cases} 
1 + 0.0916\text{Re}_p & \text{for } 0.1 \leq \text{Re}_p < 5 \\
1 + 0.158\text{Re}_p^{2/3} & \text{for } 5 \leq \text{Re}_p < 1000 
\end{cases}$$

(7)

This correction factor $f$ was first checked by the present numerical simulation for $\text{Re}_p$ from 10 to 200. The simulated drag coefficient for the particle attached on the flat surface was then compared with the empirical drag coefficient, Eq. (3). The correction factor $f$ in Eq. (7) was then proposed to correct for O’Neill’s analytical equation to obtain the drag coefficient for the particle attached on the flat surface as

$$C_d = 1.7009 f \frac{24}{\text{Re}_p}$$

(8)

The accuracy of this equation was examined by the present numerical simulation in the range of $\text{Re}_p$ from 0.1 to 250. The effect of plate Reynolds number, $\text{Re}_x$, on $C_d$ was also studied.

NUMERICAL METHOD

In order to obtain the drag force acting on the spherical particle attached on the flat surface, a 3-D numerical simulation was conducted in this study. The computational domain is a rectangular box as shown in Fig. 1. A uniform free stream velocity $u$ was set at the inlet boundary and the spherical particle of 100 $\mu$m in diameter was situated at the bottom at a distance $x$ downstream of the inlet boundary, in which $u = 5$ to 144 m/s and $x = 55$ mm for high $\text{Re}_x$ range of 18,310 to 527,000, and $u = 0.2$ to 74 m/s and $x = 6.5$ mm for low $\text{Re}_x$ range of 87 to 32,000. Non-slip boundary condition was used on the particle and the bottom surfaces, while the prescribed pressure boundary of 101,325 Pa (1 atm) was applied on the remaining surfaces.

The governing equations are Navier-Stokes and continuity equations. Steady-state laminar incompressible flow was assumed. The Navier-Stokes and continuity equations were solved by using the STAR-CD 3.22 code (CD-adapco Japan Co., LTD) which is based on the finite volume discretization method. The pressure-velocity linkage was solved by the SIMPLE (semi-implicit method for pressure linked equation) algorithm (Patankar, 1980). The QUICK (quadratic upstream interpolation of convective kinematics) scheme was used to discretize the convection terms of Navier-Stokes equation. Hexahedral cells, which allows for finer grid spacing near the wall, were generated by an automatic mesh generation tool, TrueGrid (version 2.1.0, XYZ Scientific Applications, Inc.). The total number of grids used is 425,000 in the calculation domain. It was found that increasing the number of grids to 770,000 only resulted in a less than 1.2% difference in the $C_d$ when the particle Reynolds number was set at 250. Therefore, to save the computation time, a fixed grid number of 425,000 was used in this study. The convergence criterion of the flow field was set at $10^{-7}$ for the summation of the residuals.

The upper bound of the particle Reynolds number was set at $\text{Re}_p = 250$ since the flow over the particle becomes unsteady when $\text{Re}_p > 250$ as observed by Mochizuki (1961), while the upper bound of the plate Reynolds number was set at about 500,000 when the transition takes place from the laminar to turbulent flow as reported by Munson et al. (2006).

Fig. 1. Schematic diagram of the computational domain for the flow over a spherical particle attached on the flat surface.
Sweeney and Finlay (2007) found that the drag coefficient is dependent on the plate Reynolds number. In their study, $Re_x$ was increased from 32,400 to 500,000 at a fixed $Re_p$ of 250 only and the drag coefficient was found to increase by 17.4%. In this study, the influence of $Re_x$ was extended to different $Re_p$ from 3 to 250.

RESULTS AND DISCUSSIONS

The present simulation was first validated by checking the drag coefficient for a particle suspended in the air. As shown in Fig. 2, the calculated drag coefficient is seen to agree well with Eq. (6) with the deviation smaller than 1.3–5.6% for $10 \leq Re_p \leq 200$, with the smallest error of 1.3% at $Re_p = 200$. It is seen that the numerical method is able to predict the drag coefficient very well.

For the spherical particle attached on the flat surface, Fig. 3 shows the comparison of $C_d$ obtained from the present numerical simulation with that in Eq. (8) and previous studies at low $Re_x$ range from 87 to 32,000. The simulated drag coefficient compares very well with the value of Eq. (8) and the empirical value of Sweeney and Finlay (2007), Eq. (3), for $Re_p = 0.1–250$. O’Neill’s equation is valid only at low particle Reynolds number of less than 1.0 as expected. The drag coefficient of Eq. (8) is slightly higher than the present simulation because of the $Re_x$ effect, which was found by Sweeney and Finlay (2007) at $Re_p = 250$ and will be elucidated further in the following discussion. In comparison, both O’Neill’s equation and Eq. (5) used by Ibrahim et al. (2008) deviate from the present simulation very much when $Re_p > 1.0$.

As discussed previously, $Re_x$ has an effect to increase the drag coefficient and such effect was investigated by increasing the $Re_x$ or increasing the distance $x$ at a fixed $Re_p$. In the high $Re_x$ range of this study, $x$ was fixed at 55 mm as shown in Fig. 1. When $Re_p$ is increased, the corresponding $Re_x$ will also be increased with the maximum value of $Re_x = 527,000$ when $Re_p = 250$. Fig. 4 shows the comparison of $C_d$ of the present simulation with Eq. (8) at higher $Re_x$ from 18,310 to 527,000 (corresponding $Re_p = 2.8–250$), and low $Re_x$ from 87 to 32,000 (corresponding $Re_p = 0.1–250$). In the high $Re_x$ range, when $Re_p$ is 250, $C_d$ of Eq. (8) is in very good agreement with the present numerical simulation results at $Re_x = 527,000$ ($Re_p = 250$) with the deviation of only 1.6%, which confirms the finding of Sweeney and Finlay (2007). At $Re_p$ of 91 and $Re_x$ of 256,320, $C_d$ is also increased and the deviation is increased to 4.7%. When $Re_p$ is less than 91, $Re_x$ can’t be

![Fig. 2. Comparison of the calculated drag coefficient for a particle suspended in the air with the empirical values.](image1)

![Fig. 3. Comparison of the drag coefficient of the present simulation with Eq. (8) and previous studies at low $Re_x = 87–32,000$.](image2)

![Fig. 4. Comparison of the drag coefficient of the present simulation with Eq. (8), effect of $Re_x$.](image3)
increased too much since the distance $x$ will have to be extended to an unreasonable large number. Under this condition, Eq. (8) will overestimate the drag coefficient as shown in Fig. 4 but the maximum overestimation is less than 10.7% in the high $Re_x$ range.

To find the applicable range of $Re_x$ for Eq. (8), the effect of $Re_x$ on $Cd$ at $Re_p = 250$ was studied. As shown in Fig. 5, the simulated drag coefficient increases and approaches Eq. (8) as $Re_x$ is increased from 20,000 to 527,000. The deviation is decreased from 22.7 to 1.6%. The reason why $Cd$ decreases with decreasing $Re_x$ can be seen in Fig. 6 where boundary velocity profiles at $Re_x$ of 32,000, 145,000, and 527,000, respectively, at $Re_p = 250$, are shown. It shows that velocity profile at the top half of the particle becomes steeper as $Re_p$ is decreased from 527,000 to 32,000. This means that the velocity on the top half of the particle is reduced leading to the reduction of the drag coefficient as $Re_x$ is reduced.

CONCLUSION

The previous analytical solution for $Cd$ for a spherical particle attached on the flat surface, which was derived by O’Neill (1968), is only valid in the creeping flow conditions. In this study, $Cd$ was calculated numerically to cover a wide $Re_p$ range of 0.1 to 250. For a particle suspended in the air, an empirical drag coefficient exists, which is defined as $Cd = f \times 24/Re_p$, where $f$ is a correction factor depending on $Re_p$. This correction factor $f$ was checked by the present numerical simulation and found to be correct. The simulated drag coefficient for the particle attached on the flat surface also agrees well with the empirical drag coefficient proposed by Sweeney and Finlay (2007) for $0.1 \leq Re_p \leq 250$. It was found that the correction factor $f$ for the suspended particle can also be used to correct for O’Neill’s analytical equation to obtain $Cd$ for the particle attached on the flat surface as Eq. (8) for $Re_p = 0.1$ to 250. At high $Re_p$ from 18,310 to 527,000, the maximum error of Eq. (8) is only 4.7% and the effect of $Re_p$ on $Cd$ is small. At low $Re_p$ from 87 to 15,580, the maximum error of Eq. (8) is less than 10.7% when $Re_p < 100$ and the effect of $Re_p$ on $Cd$ is also limited.

REFERENCES


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